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by Hari Susanto

Fuzzy modelling to predict students' attitudes toward mathematics using mathematical anxiety and self-efficacy data

H P Susanto[*] and T Sutarti

STKIP PGRI Pacitan, Pacitan, Indonesia

*haripurnomosusanto@gmail.com

Abstract. This paper discusses how to predict students' positive attitudes toward mathematics using mathematical anxiety and self-efficacy data. The dimension is rooted from many researchers concluding that mathematical anxiety, self-efficacy and positive attitudes towards mathematics have relationship each other. We also modelize form to predict students' attitudes using a fuzzy model. 312 data for training and 68 data for testing models used is from 380 pairs of anxiety, self-efficacy, and positive attitudes data. Further analysis showed that the first fuzzy model within 13 fuzzy sets produces 61 fuzzy rule base and the fuzzy model with 16 fuzzy sets produces 82 fuzzy rule base. Every model is carried out by using MAPE for training and testing the data. The first fuzzy model, the second fuzzy model and regression model with training data successively produced MAPE of 7.35%, 7.06%, and 7.98%. Furthermore, using the testing data of the three models successively produced MAPE values of 7.43%, 7.16%, and 7.45%. Based on the MAPE, it can be concluded that the fuzzy model is better than the regression, and the fuzzy models with 16 fuzzy sets and 82 rules are the best models that can be employed to predict students' positive attitudes toward mathematics.

1. Introduction

Recently, the theory of fuzzy concept has developed in a variety of directions including mathematics education. In this case, some factors that influence students' mathematics learning outcomes are students' positive attitudes toward mathematics, mathematical anxiety, and self-efficacy [1]. Mathematical anxiety has a negative correlation with students' positive attitudes toward mathematics and vice versa in junior high school students and university students [2-4]. Akin and Kurbanoglu uncovered that a positive attitude has a positive correlation with self-efficacy; conversely, a negative attitude has a negative correlation with self-efficacy [4]. Then, based on those three variable correlations, the present paper will uncover best mathematical modeling for predicting students' attitudes toward mathematics using data anxiety and self-efficacy.

Regression is a statistical model commonly used to determine the correlation between two or more variables and to predict a dependent variable value. Four assumptions must be fulfilled in the regression model those are 1) linearity of the phenomenon measured 2) constant variance of the error terms 3) independence of the error terms 4) normality of the error term distribution. If those four assumptions have been satisfied, then the regression model will have a sharp accuracy to determine correlations and predictions; on the contrary, if one of them is not fulfilled; consequently, the regression model will provide inaccurate explanations and predictions [5].

The fuzzy model is a mathematical model that is not bounded by classical assumptions in its application for a data prediction. This model provides a convenient process and genuine logic towards the more tangible variable characteristics. In other words, this model is a certainty-based theory. The significance of the fuzzy can model data that is only based on expert opinion as well as empirical data-based modeling [6].

Fuzzy model is a system which consists of several inseparable components, and those are fuzzifier, fuzzy inference engine, defuzzification [7]. The main element to be the brain for deciding the results is the fuzzy inference engine. In this section, there is a logic containing rules that are formed based on theory, expert opinion or empirical data.

Based on the fuzzy system components, the fuzzification process in the present article employs singleton fuzzifier, multiplication inference engines [7]. Further, the defuzzification process applies the center average defuzzification and the Gaussian membership function. The rules on the fuzzy inference machine are formed using empirical data. Then, the selection process is carried out using the lookup scheme table [7]. Abadi shows that the fuzzier sets are defined in dependent and independent variables; the smaller predictions of MAPE it produces. Furthermore, the formed fuzzy model will be compared with the regression model [8].

2. Method
In this article, there are two independent variables namely mathematical anxiety and self-efficacy, while the most significant variables are students' attitudes toward mathematics. The data collection process was carried out on 380 students from high schools and vocational high schools in Pacitan sub district. The 312 data is used for training data, while the rest is used for the data of the formed testing model.

2.1. Regression
Let n be given pairs of observation data on the dependent variable Y and p predictor or independent variables $x_1, x_2, x_3,...,x_p$. Linear regression model is written as the equation for [9,10].

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \varepsilon \tag{1}$$

Where β_0 Constan, $\beta_1,\beta_2,...,\beta_p$ coefficient from predictor, and ε error from prediction.

A fuzzy system is a basic knowledge or basic rule system which consists of IF-THEN Fuzzy rules [7]. Four prominent components for processing input and output are fuzzy rule bases, fuzzy inference engines, fuzzifiers, and defuzzifiers. The fuzzy system formation procedure can be seen in figure 1 below.
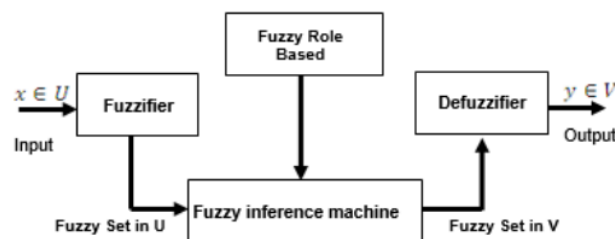

Figure 1. Fuzzy system.

The process of forming a fuzzy rule base employs a lookup scheme table carried out in the following steps [7].

Let N be given pairs of training data from self-efficacy data (x1), anxiety (x2), and attitude (y) students towards mathematics: $(x_{1i}, x_{2i}; y_i)$ for $(i = 1,2,3,....,n.)$

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2.1.1. Step 1. Define the universal set of input and output domains. In this process, the tolerance range of each data will be determined x_1, x_2, and y by determining the minimum and maximum value of the training data used.

2.1.2. Step 2. Define the fuzzy set on a continuum, normal and complete universal set.

2.1.3. Step 3. Form one fuzzy rule for one pair of training data and obtained fuzzy rules:

$$A_i^l, B_i^l, C_i^l \qquad (2)$$

2.1.4. Step 4. Calculate the degree of each fuzzy rule obtained from step 3. If there are conflicting fuzzy rules, then select the highest degree of fuzzy rules for each data pair.

2.1.5. Step5. Form Base of fuzzy rules obtained from step 4.

$$D(rule) = \prod_{i=1}^{n} \mu_{A_i^l}(x_{1i}) . \mu_{B_i^l}(x_{2i}) . \mu_{C_i^l}(y_i) \qquad (3)$$

2.1.6. Step 6. The form of a fuzzy model consists of fuzzy rule bases, fuzzification, fuzzy inference engines, and defuzzification. In this article, a fuzzy model is constructed with singleton fuzzification, multiplication inference engine, center mean defuzzification and Gaussian membership function, namely.

$$y_i^* = f(x_{1i}, x_{1i}) = \frac{\sum_{j=1}^{m} y_j^{*l} . \exp(-\left(\frac{(x_1 - x_{1j}^{*l})^2}{\sigma_1^2} + \frac{(x_2 - x_{2j}^{*l})^2}{\sigma_2^2}\right))}{\sum_{j=1}^{m} \exp(-\left(\frac{(x_1 - x_1^{*l})^2}{\sigma_1^2} + \frac{(x_2 - x_2^{*l})^2}{\sigma_2^2}\right))} \qquad (4)$$

Note that:
y_i^* = the value of the students' attitudes predictions to mathematics to-i
m = the number of formed fuzzy rules
y_j^{*l} = center of the fuzzy set C output in the rule to-j
x_1^{*l} = center of the fuzzy set A input to the rule to-j
x_2^{*l} = center of the fuzzy set B input on the rule to-j
σ_1 = support for each A fuzzy set
σ_2 = support for each B fuzzy set

3. Results and discussion
In this section, we will discuss the application of fuzzy models to predict students' attitudes toward mathematics by using anxiety and student self-efficacy data. The two formed fuzzy models in this article have students' anxiety and self-efficacy for data input as well as students' positive attitudes as their outputs. The process of forming the second fuzzy model is carried out by using the following scheme lookup table.

312 pairs of training data are given from Mathematical Anxiety and Self-Efficacy as well as students' positive attitude towards mathematics variables.

3.1. Step 1
Each variable in the input and output is defined as a universal set (12.5 77.5) for model 1 and (12 76) for fuzzy model 2.

3.2. Step 2
Each variable of model 1 and model 2, is respectively defined by the fuzzy set as much for 13 and 16. Support each fuzzy set on model 1 that is 5, and on model 2, that is 4.

3.3. Step 3
In this step produce 312 rules.

3.4. Step 4
By reducing the rules of step3 using D (rule) obtained 61 rules for model 1 and 82 rules for model 2.

3.5. Step 5
Fuzzy rules based are formed 61 rules for model 1 and 82 rules for model 2.

3.6. Step 6
The fuzzy model which is formed is footnotes should be avoided whenever possible. If required they should be used only for brief notes that do not fit conveniently into the text.

3.6.1. Fuzzy model-1

$$y_i^* = f(x_1, x_2) = \frac{\sum_{j=1}^{61} y_j^{*l} . \exp\left(-\left(\frac{(x_1 - x_{1j}^{*l})^2}{5^2} + \frac{(x_2 - x_{2j}^{*l})^2}{5^2}\right)\right)}{\sum_{j=1}^{61} \exp\left(-\left(\frac{(x_1 - x_{1j}^{*l})^2}{5^2} + \frac{(x_2 - x_{2j}^{*l})^2}{5^2}\right)\right)}$$

3.6.2. Fuzzy model-2

$$y_i^* = f(x_1, x_2) = \frac{\sum_{j=1}^{82} y_j^{*l} . \exp\left(-\left(\frac{(x_1 - x_{1j}^{*l})^2}{4^2} + \frac{(x_2 - x_{2j}^{*l})^2}{4^2}\right)\right)}{\sum_{j=1}^{82} \exp\left(-\left(\frac{(x_1 - x_{1j}^{*l})^2}{4^2} + \frac{(x_2 - x_{2j}^{*l})^2}{4^2}\right)\right)}$$

Further, the training data is also used to create a model to predict the value of students' positive attitudes using a regression model. Regression models which are formed using training data have met all the parametric assumptions required. The regression model of the training data has similarities,

$$y^* = 10{,}78034 + 0.08338x_1 + 0.78543x_2$$

Comparison of those three models is carried out by calculating the Mean Square Error (MSE) and Mean Absolute Percentage Error (MAPE). The smaller the MAPE value being produced, the better the model is. The results of MSE and MAPE calculations from the three models are written in the table 1,

Table 1. MSE dan MAPE of each model with training data.

Comparator	Regression Model	Fuzzy Model -1	Fuzzy Model -2
MSE	22.58	21.11	18.97
MAPE	7.98%	7.35%	7.06%

Table 1 shows that the regression model has a larger MSE compared to the two formed fuzzy models; likewise, the fuzzy model 1 has a larger fuzzy MSE compared to the fuzzy model 2. It means that the predictive value in the training data of fuzzy model 2 has a closer prediction of students' attitudes to the original value compared to the regression model and fuzzy model 1. Furthermore, the smallest MAPE value is owned by the fuzzy model 2. It reveals that the fuzzy model 2 is the best model for predicting students 'attitudes with students' mathematical anxiety and self-efficacy input. More specifically, the comparison of actual and predictive data of each model can be seen in the figure 2.

The 312 pairs of training data, used to form the regression and fuzzy models above, is obtained to predict the students' attitude data. Additionally, the model testing is done using 68 data testing's. The results of each model testing using the testing data can be seen in table 2.

Table 2. MSE dan MAPE of each model with testing data.

Comparator	Regression Model	Fuzzy Model -1	Fuzzy Model -2
MSE	22.49	23.42	20.78
MAPE	7.45%	7.43%	7.16%

Based on MSE and MAPE in table 2, it is found that the fuzzy model 2 has the smallest MAPE compared to both models. In essence, the fuzzy 2 model is the best model if used to predict students' attitudes in the population. For more details, the prediction of the testing data using the three models can be seen in the figure 3.
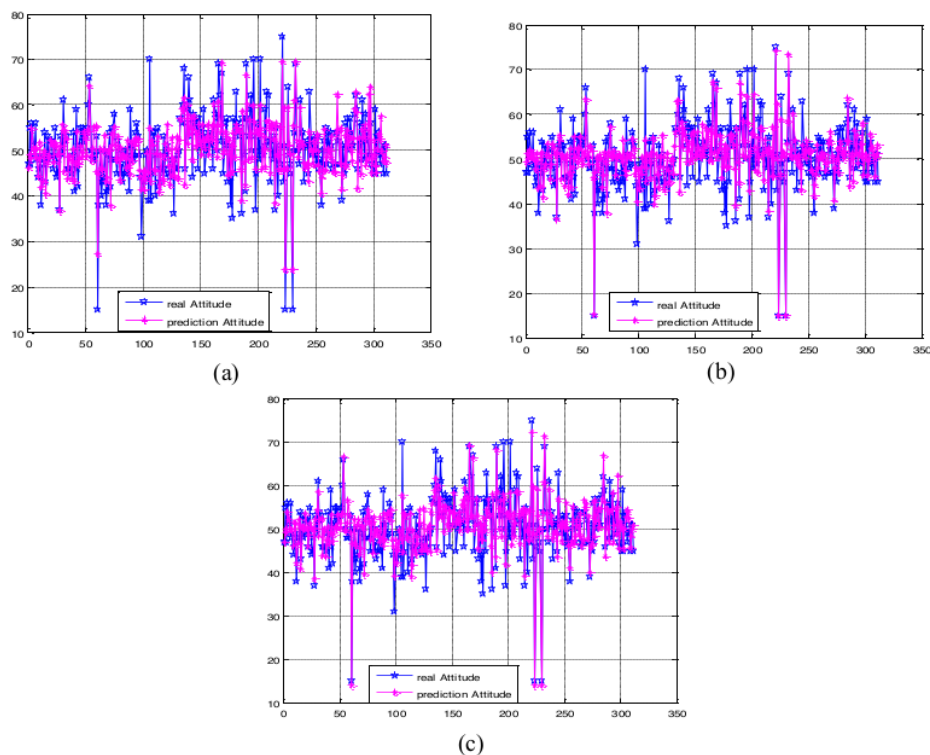

Figure 2. Realand predictive data of attitude training data, (a) regression models, (b) fuzzy model-1, (c) fuzzy model-2.
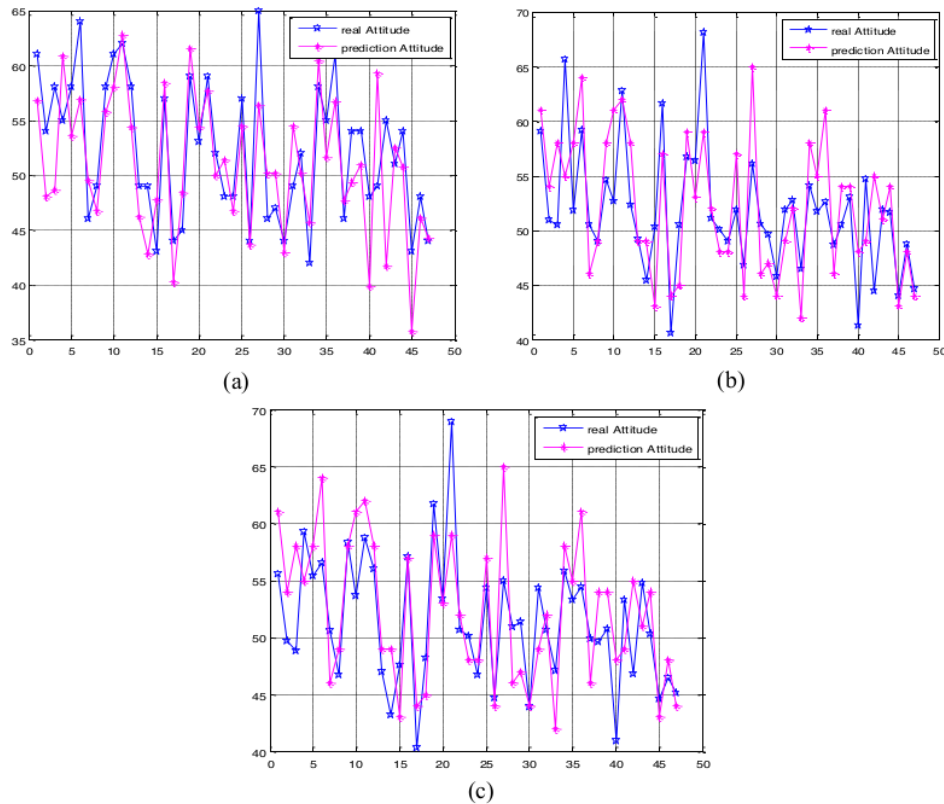
Figure 3. Realand predictive data of attitude testing data, (a) regression models, (b) fuzzy model-1, (c) fuzzy model-2.

Table 1 and table 2 show that second fuzzy model is better than the fist fuzzy model, because the number of fuzzy set that difine on the second step of second fuzzy model is more than fuzzy set in fuzzy model-1 [11]. Furthermore,The second fuzzy model is better than linier regression model in predicting students attitude Kushwaha and Kumar conclude that fuzzy methods have more advantage over statistical analysis to control variation in data of behavioral research [12]. The advantage of fuzzy model are reducing uncertainty on the raw data and it enables the use of uncertainty measures to quantify the ambiguity associated with prediction of psychological parameters [12].

Based on the models used outside the field of psychology, fuzzy models also have better results than the regression models in predicting. Gokulachandran and Mohandas found that fuzzy model is capable of giving better predictionof tool life and more acurate in predicting than regression model [13]. Minh, Katushin, Antonov, and Veinthal found that the fuzzy logic model has provided the best accurate output predictions dealing with the uncertain inputs to predict tunnel boring machine rate of penetration [14]. Kovac, Rodic, Pucovsky, Savkovic, and Gostimirovic conclude that the fuzzy logic modeling technique can be effectively used for the prediction of surface roughness in dry machining [15].

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4. Conclusion

Fuzzy model 2 is the best model if used to predict students' attitudes in non-respondent students in a population. The more sets defined in step 2, the better model fuzzy will be obtained. The fuzzy model does not require assumption tests like in the regression model.

In the next research, it would be better to uncover the following points, namely, 1) to determine the way to find the best fuzzy model shortly, like Stepwise in the regression model. 2) Determining fuzzy base rules using complete rules [11], in the fuzzy model -1 and fuzzy model-2 will obtain 169 rules and 256 rules if a complete fuzzy base rule is created. Complete fuzzy base rules will provide more general scope for the population used.

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